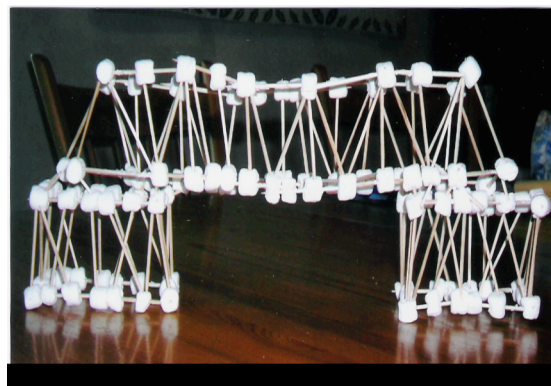
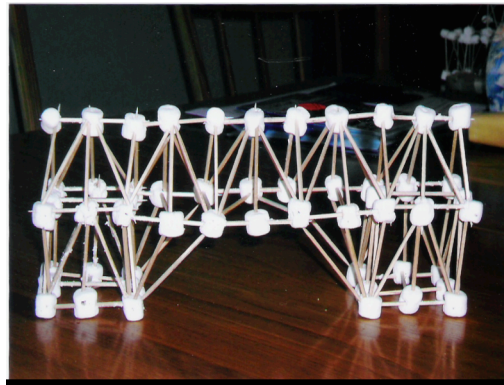
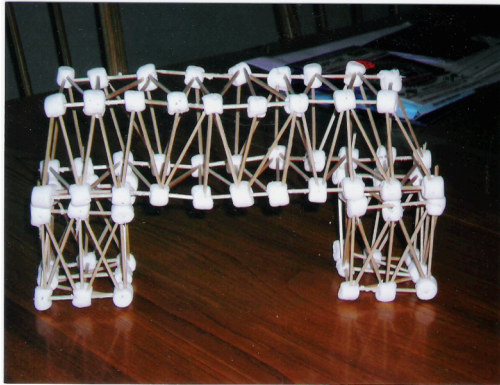


Bridge Unit

Introduction

The idea of constructing a bridge from toothpicks, to a specific budget and criteria, has been around for many years and has been done from 5th grade classes up through college classes. If you google 'toothpick bridges' on the internet, you will find more material than you will want to spend time reading! Not much of this material, however, will work well in a typical adult education class. In these lesson plans I try to break down the design process into small steps, and explicitly incorporate and explore related math concepts and skills. I do not grade the final bridge (there are many rubrics that you can find on the internet that do so) because I have graded each step along the way and I am more interested in the students' efforts.

Below are pictures of the three bridges the students built. There were large differences in quality and strength. (The second bridge is not to code.) After the bridges were built they were weighed, then broken to see how strong they were. These bridges weighed from 1.6 to 2.7 ounces, and supported weights ranging from 3 to 19 pounds. A bridge efficiency ratio was calculated from this. This was a lot of fun and I think that most of the students enjoyed at least parts of the process.



Notes on materials

The most difficult materials to procure are the scales to weigh the bridge and the weights used to break the bridges. You need to be able to weigh the bridges to roughly quarter ounce intervals at least, and add weights to the bridge in roughly one pound increments. If there is money available, Staples sells a postal scale for \$40 which weighs to the nearest tenth of an ounce, upwards to five pounds. Does your adult ed office have a postal scale you can use? If not, dieters scales (used to weigh food) are cheaper.

To break the bridge, place a basket on top and add weights in roughly one pound increments. Use your imagination here. I had ten, 1 pound exercise weights. After that, I stacked GED books on top of the basket. Soup cans could work as well. If the scale you used for the bridges does not go beyond one pound, use a bathroom scale to weigh the books/cans/whatever before piling it on the basket.

Notes on teaching percents

There are problems where the students are to calculate the percent of a number. The percent was 'easy' in that it ended in a 0 or 5. Some of the students like to use a calculator but many preferred to do it in their head, working from either 10 or 25 percent. Encourage students to be comfortable both ways.

Overview of lesson plans

The students first learn to construct orthographic drawings using children's building blocks. They will measure in centimeters and their drawings must be to scale (a very simple scale). In the second lesson the students will experiment with toothpicks and minimarshmallows, and build three structures that are potential tower designs (they do not need to be significantly different from each other). They will draw scaled, orthographic drawings of these three structures. Next, the students work out specific calculations associated with the bridge, and create and draw two potential roadway designs. The calculations involve problem solving, percents, and interpreting ratios. In the fourth class, they select the tower and roadway designs they will use, figure out how to connect the roadway to the towers, draw the bridge drawings, and estimate if they are in budget. In the fifth class they build the bridge. In the sixth class the students will take a test and break the bridges.

You can cut this down to 5 classes by eliminating most of the work in the fourth class. They would still need to do the first four problems of that night's assignment (pick the tower and roadway they want and figure out how they will attach together). The rest of that night's worksheet is creating orthographic drawings of the entire bridge, and making estimates on the number of toothpicks and marshmallows needed, and estimating if they are in budget.

Class #1, Orthographic Drawings

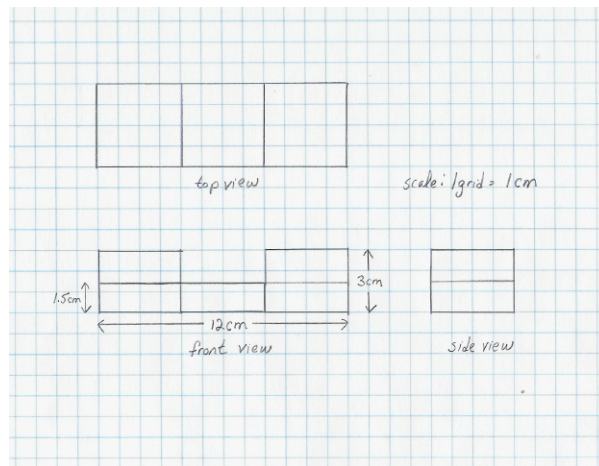
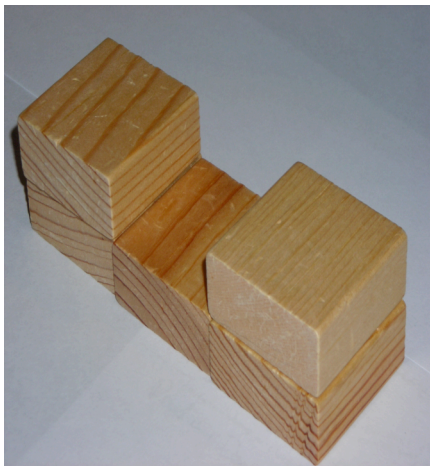
Materials: bag of children's building blocks

graph paper

centimeter rulers

box of pencil erasers – the kind that go on the end of the pencil

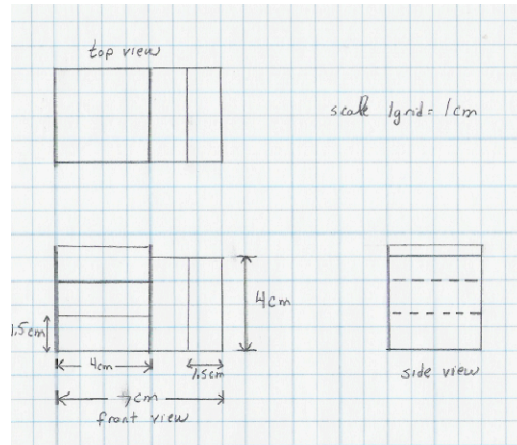
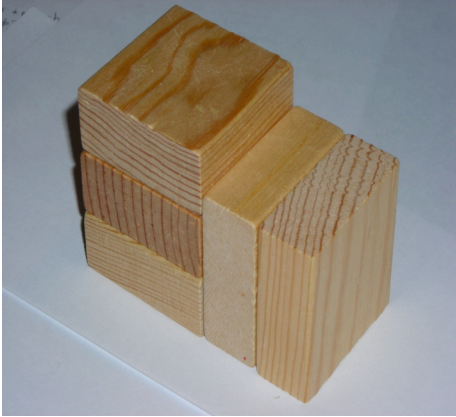
Orthographic drawings are the standard method used in industry to convey information about parts and structures. They consist of a front, side, and top view of the part. The students will learn the basics from building blocks, then later (in another class) apply the skill to their bridges. Start with a simple structure built from children's building blocks, such as pictured below:



Have the students note the following details:

- The scale is 1grid = 1cm (actual dimensions are marked, as is customary in orthographic drawings, but the students will not be required to do so – making the drawing to scale will be enough)
- The views are aligned with each other. The front view is directly underneath the top view, and the edges are aligned (meaning the front view is not to the right or left of the top view, but directly underneath). The side view is to the right (or it can be to the left) of the front view, and its edges are also aligned (the side view is not higher or lower than the front view, but directly across).
- Each solid line in the drawing is a line that you see as you look at the blocks.

Next, try a more complicated structure such as:



Have the students note the following:

- The side view is a bit tricky to understand. A solid line is a line that you can see. A dashed line means that is a line that you cannot see. As you look at the structure from the right side, you see a line at the top of the two 'standing up' blocks; this is the second solid line from the top. You do not see the two lines that are created by the pile of three 'lying flat' blocks; that is why they are dashed. (All lines on their bridge drawings will be visible; there should be no need for dashed lines. However, these drawings are so prevalent in industry, that it is worthwhile to introduce students to these ideas in this class.)

Set up three or four stations, each with a different structure. Put the students in groups and have them construct drawings for each station. Check their work as they go along, or, have answer keys for each station. Lastly, you can have each group create their own structure and orthographic drawings. They can switch the drawings with another group's, and try to build the structure.

Grading

You can grade the student's work as it is completed during class. A sample grade sheet could look like:

Stock Market (10 points) _____
 Notes from structure #1 (2 points) _____
 Notes from structure #2 (2 points) _____
 Drawings from station #1 (10 points) _____
 Drawings from station #2 (10 points) _____
 Drawings from station #3 (10 points) _____
 Built structure (6 points) _____
 Total (50 points) _____

Homework:

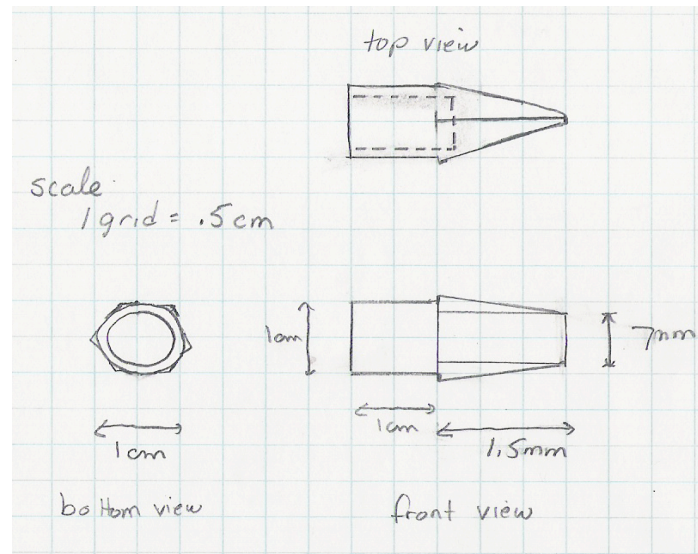
You will need a box of pencil erasers – the kind that go on the end of the pencil. These are inexpensive, so each student can have one to take home, and are fairly simple to draw. Pass out an eraser and graph paper to each student, along with the following directions:

Using graph paper, construct a top view, side view, and bottom view of the eraser. The bottom view is where the hole is in the eraser. Measure in centimeters. Use the scale 1 grid = .5 cm (which is the same as every 2 grids = 1 centimeter). This will make the drawing larger than actual size.

If you do not have a centimeter ruler at home, jot down the dimensions before you leave class.

There are only a few measurements that you need.

Answer key:



Students do not need to write in dimensions, but drawings should be to scale.

Class #2, Tower Design

Materials: sample bridge (to show students)

- toothpicks
- mini marshmallows
- matchbox car
- graph paper
- centimeter rulers
- starburst candy
- plastic grocery bags (one for each group)

The students will be introduced to the bridge and design criteria, and design the tower part of their bridge. Hold up a sample bridge – do not let the students look at it too long. They need to have some idea of what they will be doing, but you do not want them to copy your bridge. Explain that at the end of the project the bridges will be broken to see how much weight they can hold. A basket will be placed on the top of the bridge, and weights will be slowly added, until the bridge breaks. However, the students will not be able to make the bridge strong by simply using a lot of toothpicks; they have to stay within budget. Pass out and go over the bridge criteria.

Bridge Criteria

1. The bridge must be a minimum of 5 centimeters high (from the table to the bottom of the bridge).
2. The span of the bridge must be a minimum of 15 centimeters long (this is the distance between towers).
3. The bridge must be wide enough for a matchbox car to ride across.
4. The total budget for the bridge is 10 million dollars.
 - a. The toothpicks come in bags of 10. One bag costs \$245,000.
 - b. The marshmallows come in bags of 10. One bag costs \$205,000.
 - c. The labor is 125% of the cost of material.
5. The bridges will be weighed, then broken to determine their maximum load. The

bridge efficiency will be calculated. The bridge efficiency = $\frac{\text{load}}{\text{weight}}$.

The details of the budget and bridge efficiency will be explored in later classes. Tonight they are to come up with three tower designs. They will make a decision about which design to use in a later class. Put the students in groups, and pass out the assignment.

Tonight's Assignment

Tower design

Play around with the marshmallows and toothpicks and share your ideas. You can come up with as many tower designs as you would like, but you must pick three for your drawings.

Drawings

- create 3 separate set of drawings, one for each design
- for each design create a front, top and side view
- use the scale 1 grid = 2 cm
- for each design write number of toothpicks and number of marshmallows used
- write on the drawing any comments or thoughts you might note (such as this does not sit square, this is lopsided, this seems solid, etc.)

Think about

- what shapes do you think are strong?
- what shape do you want for the bottom, for the sides?
- where do you think this will fail under load?
- remember must be at least 5 cm high and wide enough for a matchbox
- remember also you are under budget constraints

Each group will have the same three designs, but everyone is to have their own drawings.

At the end of the evening, place your toothpick structures in a plastic bag along with your groups' names written on a piece of paper. Pass in your drawings to be graded.

NOTE: The structures will most likely warp as the marshmallows dry. Readjusting the structures once or twice within the first twenty-four hours will minimize this.

Grading

Unless you are very efficient, you will probably not be able to grade all of the drawings during class time, so the students will get their grade sheet the following week. A sample sheet could look like:

Stocks (10 points) _____

Drawings for design #1 (10 points) _____

Drawings for design #2 (10 points) _____

Drawings for design #3 (10 points) _____

Building 3 structures (10 points) _____

Total (50 points) _____

Homework:

The students will create structures and drawings at home. Starbursts are not as messy as marshmallows (they travel better) and will not warp. You can buy them in large bags, which is cheaper than buying individual packages. Pass out graph paper, 15 toothpicks and 8 starbursts, along with the following directions, to each student.

Create any type of three dimensional structure you want. (This means that the structure cannot be flat – it must stand upright.) It does not have to be anything like the tower design. It can be completely nonsensical if you like. You must use a minimum of 10 toothpicks and 6 starbursts. Create a front, top, and side view drawing of your structure (do not draw the starbursts, only the toothpicks). Use the scale 1 grid = 2 cm. Pass in your drawings and structure.

If you do not have a cm ruler at home, jot down the measurements of a toothpick and starburst, and estimate the size of your structure.

Note: When reviewing the homework, you could, for fun, create a quick overhead of one of the student's set of drawings, and see if the rest of the class could build the structure.

Class #3, Calculations and Roadway Design

Materials:

Calculators

same as second class (minus the starburst)

The students will work through calculations involving the bridge budget and bridge efficiency ratio. They will then work on their roadway design. Put the students in groups and pass out the following worksheet. See how much the students can do on their own before helping.

Bridge Calculations

Name: _____

1a. You have built a bridge using 82 toothpicks and 63 marshmallows. The toothpicks come ten to a bag and cost \$285,000 per bag. The marshmallows come ten to a bag and cost \$225,000 per bag. The labor cost is 90% of your materials cost. What is the total cost of your bridge?

1b. The budget for the bridge was 8 million dollars. How much under budget was the bridge? What percent under budget was the bridge? Do you think this leaves enough room for unforeseen problems?

1c. Boston's Big Dig was one of the nation's most expensive and poorly managed public works projects. It was estimated to cost 4 billion dollars (in today's dollars), and finished at 14.6 billion dollars. How much over budget was the project? What percent over budget was it?

2. The bridge efficiency is calculated as the $\frac{\text{maximum load the bridge can hold}}{\text{Weight of the bridge}}$

a. A bridge can hold 83 ounces, and weighs 1.7 ounces. What is the bridge efficiency? (round to the nearest tenth)

b. A bridge holds 5 pounds, 3 ounces, and weighs 1.2 ounces. What is the bridge efficiency? (there are 16 ounces in one pound)

c. A bridge hold 5 pounds, 3 ounces and weighs 1.8 ounces. What is the bridge efficiency?

d. Which of the three bridges, a, b, or c is the best bridge? Why?

e. Is it better to have a high or a low bridge ratio?

f. In your own words, what does the bridge efficiency ratio tell us? Why would engineers want to make this calculation?

Answer Key:

1a. 9 toothpick bags = \$2,565,000 7 marshmallow bags = \$1,575,000

Total material cost \$4,140,000

Labor : 90% of material is \$3,726,0000

Total cost is labor + materials = \$7,866,000

1b. \$134,000 under budget

$134,000/8,000,000 \times 100 = 1.7\%$ under budget (to find the percent over or under budget you take the difference, and divide it by the original budget)

1c. $14.4\text{billion} - 4 \text{ billion} = 10.6 \text{ billion}$ $10.6/4 \times 100 = 265\%$ over budget

2. a. 48.8 b. 69.2 c. 46.1

d. Bridge b is the best. All three bridges can support the same weight (83 ounces), but bridge b weighs the least, implying that bridge b is constructed with less material. This implies that there must be something superior to bridge b's design, because it can support the load with the least amount of material (assuming all the bridges have the same span).

e. a high bridge ratio

f. The bridge efficiency ratio compares how strong the bridge is to the amount of material used. If a bridge is very strong, but needs a lot of material (weight) to be that strong, then it will have a low bridge ratio. It tells how many times its weight the bridge can hold.

Next, the students, are to create and draw orthographic drawings for two roadway designs. Put the students in their groups and pass out the following:

Tonight's Assignment

Roadway Design

1. Play around with the marshmallows and toothpicks and share your ideas. You can come up with as many roadway designs as you would like, but you must select two for your drawings.

Drawings

- create two separate set of drawings, one for each design
- for each design create a front, top and side view
- remember, the top view is to be on top of the front view, and the side view to the side of the front view
- use the scale 1 grid = 2 cm
- for each design write number of toothpicks and marshmallows used
- write on the drawing any comments or thoughts you might note

Think about

Where do you think this will fail? Use more toothpicks here. Remember the toothpicks will now be sideways, which is the direction they are the weakest. The weights will be placed on the bridge from the top. How can you make this strong?

Each group will have the same two designs, but everyone is to have their own drawings.

At the end of the evening, place your toothpick structures in a plastic bag along with your groups' names written on a piece of paper. Pass in your drawings to be graded.

Grading

A sample grade sheet could look like:

Stocks (10 points) _____

Calculations worksheet (20 points) _____

Drawings for design #1 (10 points) _____

Drawings for design #2 (10 points) _____

Total (50 points) _____

Homework

The students will practice bridge calculations on the following worksheet:

Answer Key to homework:

1. Total cost \$10,474,000

2. $474,000/10,000,000 \times 100 = 4.7\%$

3. 45 and 41.3 The first bridge is better because the ratio is higher. It holds more weight using less material, when compared to the other bridge.

4. It has a more efficient design. The design is stronger. It holds 45 times its own weight and the other bridge only holds 41 times its own weight.

Homework

1. You have constructed a bridge made of toothpicks and marshmallows. You have used 74 toothpicks and 58 marshmallows. The toothpicks come 10 to a bag and cost \$305,000 per bag. The marshmallows come 10 to a bag and cost \$280,000 per bag. The labor is 95% the cost of the material. Calculate the total cost of your bridge.

2. The budget for the bridge was \$10 million. What percent under/over budget are you? Round to the nearest tenth.

$$\text{Bridge Efficiency Ratio} = \frac{\text{maximum load the bridge can hold}}{\text{weight of the bridge}}$$

3. The first bridge holds 90 ounces and weighs 2 ounces.
The second bridge holds 95 ounces and weighs 2.3 ounces.
Which is the better bridge? Why?

4. What do I mean by 'better bridge'?

Class #4, Finish bridge design and create bridge drawings

Materials: same as class #3

The students will pick the tower and roadway design they want to use, figure out how they will connect the roadway to the towers, estimate the total number of toothpicks and marshmallows needed, determine if they are on budget, and create their final bridge drawings.

Tonight's assignment
Completing the bridge design

1. With your group, pick the tower design and roadway design you want to use. You may use the ones from the previous classes, or, you can make last minute modifications. In few sentences describe why you chose the designs you did. Include in your answer comments about how the cost and strength of the structures you chose compares with the designs you did not use.

2. Tower design chosen (drawings #1, 2 or 3) _____

3. Roadway design chosen (drawings #1 or 2) _____

4. You need to decide how you will attach the roadway to the two towers. Play around with the toothpicks and marshmallows and decide on a method.

Note to teachers : Ideally, the students will realize that the roadway will be built into the towers. After they construct the towers it leads right into the roadway, so, they should not build the towers, then the roadway and try to connect it. Some students may realize this and some may not. If they do not realize this, you can either lead them to this, or, let them figure something else out. Students can build the towers and roadway separately, then take a toothpick and break it into small pieces, and use those to connect them together– the bridge can still be strong if this is done properly, although the students are using more materials.

5. Estimate the total number of toothpicks and marshmallows you will need. The bridge is to have a minimum span of 15 cm. Show your work so that I can see how you came up with your numbers.

of toothpicks needed _____

of marshmallows needed _____

6. Calculate the estimated cost of your bridge. Show all work.

The toothpicks come in bags of 10. One bag costs \$245,000.

The marshmallows come in bags of 10. One bag costs \$205,000.

The labor is 125% the cost of material

7. Are you over budget or under budget? _____

How much over or under are you? _____

What percent over or under are you? _____ Show your work.

8. If you are over budget, make some modifications. Explain what you did and recalculate the costs. Show your work.

9. Create the final drawings for your bridge. Use the same scale, 1 grid = 2cm. Do a front, top and side view. Remember the bridge is to have a minimum span of 15 cm and be a minimum of 5 cm high. Each group will have the same design but everyone will create their own drawings. Check with the other people in your group to be sure your drawings are the same.

Grading

A sample grade sheet could be:

Stocks (10 points)_____

Bridge worksheet (15 points)_____

Bridge drawings (25 points)_____

Total (50 points) _____

Homework

There is no specific assignment. If you feel that the students need more work with the bridge calculations, then create a worksheet giving them more practice. If students did not finish their classwork, they can finish it for homework. It is open for catch up, because next class the students will build the bridges.

Class #5, Building the Bridges

Materials: same as class #3

Baggies

Have lots more toothpicks and mini marshmallows on hand than you think you'll need. Some students went a little toothpick crazy here. Sort (or have the students sort) the toothpicks and marshmallows into groups of 10 and put in the baggies.

The students will build their bridges and calculate the actual cost of their bridge.

Tonight's assignment

Building the Bridge

1. You are now ready to build your bridge. You will need to keep track of the actual number of toothpicks and marshmallows used. (It may, or may not, be close to your estimate.) Every time you open a new bag, put a hash mark down (tttt) to keep track.

Toothpicks (hash marks) _____ total number used _____

Marshmallows (hash marks) _____ total number used _____

2. How close are the number of toothpicks and marshmallows used to your estimate?

Toothpicks: number off _____ percent over/under (show your work) _____

Marshmallows: number off _____ percent over/under (show your work) _____

3. If you are off by more than 10% on either the toothpicks or the marshmallows, where do you think you made your estimating mistake(s)? Answer in complete sentences.

4. Calculate the actual cost of your bridge. Show all work.

The toothpicks come in bags of 10. One bag costs \$245,000.

The marshmallows come in bags of 10. One bag costs \$205,000.

The labor is 125% the cost of material.

5. Are you over budget or under budget? _____

How much over or under are you? _____

What percent over or under are you? _____ . Show your work below.

6. How closely does your bridge match your drawings? Describe any place where the drawings do not match the bridge. You can use pictures.

7. Next class we will break the bridges. Take a guess at your bridges bridge efficiency ratio. (How many times its weight do you think your bridge can hold?) _____

Note to teachers

As the marshmallows dry, the bridges may have a tendency to warp. Most of this will happen within the first 24 hours, even the first 12 hours. A quick readjustment will solve this problem. Try to remember to do this a few times for the first day after making the bridges.

Grading

A sample grade sheet could look like:

Stocks (10 points) _____

Building the bridge (20 points) _____

Worksheet (20 points) _____

Total (50 points) _____

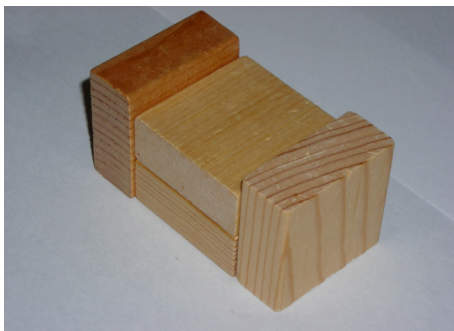
Homework

There will be a test next week. Do the following homework/study guide and study for the test.

Name: _____

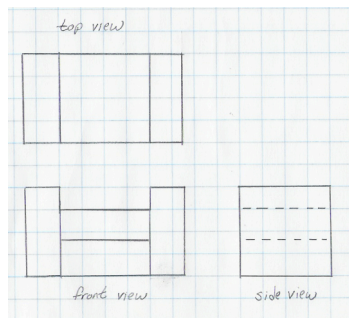
Homework/Study Guide

1. You have constructed a bridge made of toothpicks and marshmallows. You have used 84 toothpicks and 68 marshmallows. The toothpicks come 10 to a bag and cost \$310,000 per bag. The marshmallows come 10 to a bag and cost \$285,000 per bag. The labor is 105% the cost of the material. Calculate the total cost of your bridge.
2. Your budget for the bridge was \$10 million. What percent under/over budget are you? Round to the nearest tenth.
3. You estimated you would use 72 toothpicks. What percent over are you from your estimate? Round to the nearest tenth.
4. There are two bridges with the same span. The first bridge weighs 2.2 ounces and holds 11 pounds, 3 ounces. The second bridge weighs 2.6 ounces and holds 11 pounds 7 ounces. Which is the better bridge? Explain why. Use complete sentences.
5. Draw a top, front, and side view of the structure. Use the scale – the blocks are square, 4 grids wide and 4 grids long. They are $1\frac{1}{2}$ grids thick. (Also know how to construct drawings for toothpick structures.)



Answer Key

1. \$9,809,250
2. 1.9% under budget
3. 16.7% over the estimate
4. The be ratios are 81.4 and 70.4. The first bridge is better because it can hold more times its own weight.
- 5.



Class #6 – Test, and Breaking the Bridges

Materials: scales and weights to weigh and break the bridges

calculators

an ‘S’ hook

string

brush pan and brush

Breaking the bridges

This is really fun for the students. Before beginning, pass out the worksheet below and go over the first few questions with the students. It asks them to make some predictions and gives them some points to consider as the bridges break. Set up the bridges so they straddle two tables or desks, with one tower on each table. Place a basket on top of the bridge. See the notes at the beginning of the bridge section for comments about inexpensive weights and scales that you could use. The basket should be able to balance on top of the bridges if they have been built to code. It is not possible (I think) for the bridge to be wide enough to allow a matchbox car to pass and yet not be able to balance a basket on top.

Be prepared for any surprises. It is a good idea to bring an S hook and string to class also. I had one group show up without their notebooks on bridge building day. They did not build the bridge to code, so we could not rest the basket on top. The students, creatively, placed a plastic ruler down the length of the bridge and attached the S hook to the ruler (there was a hole in the middle of the ruler), then used the string to attach the basket to the S hook. While it was not perfect it did help to distribute the load evenly, so the weight did not fall onto one or two toothpicks.

Add weights in roughly one pound increments. Bring a brushpan and brush for cleanup – the bridges tend to shatter when they break.

Bridges Test

Name: _____

You must show all your work to get credit!!

1. You have constructed a bridge mad of toothpicks and marshmallows. You have used 94 toothpicks and 72 marshmallows. The toothpicks come 10 to a bag and cost \$\$295,000 per bag. The marshmallows come 10 to a bag and cost \$245,000 per bag. The labor is 110% the cost of the material. Calculate the total cost of your bridge.

2. Your budget for the bridge was \$10 million. What percent under/over budget are you? Round to the nearest tenth.

3. You estimated you would use 84 toothpicks. What percent over are you from your estimate? Round to the nearest tenth.

4. Bridge efficiency ratio = $\frac{\text{weight the bridge held}}{\text{weight of the bridge}}$

The class built 3 bridges. The first bridge weighs 2.1 ounces and held 49.3 ounces, the second bridge weighs 2.3 ounces and held 8 pounds, 3 ounces, and the third bridge weighs 2.5 ounces and held 8 pounds, 11 ounces. (There are 16 ounces in a pound.)

Which is the best bridge?

Why?

5. What does the bridge efficiency ratio tell us about the bridge? (Answer with complete sentences.)

6. On graph paper, construct a scaled, front, top, and side view of the toothpick structure. Use a 2:1 scale.

(create a toothpick structure)

7. On graph paper, construct a scaled, front, top and side view of the block structure. Use the scale 1 grid = 1 cm.

(create a block structure)

Breaking the Bridge

1. What do you think the efficiency ratio of your group's bridge will be?
2. Where do you think your group's bridge will fail? (draw a quick sketch)
3. Which do you think will break first and cause the bridge to fail – the toothpicks or the marshmallows?

Watch closely as the bridges break and see if you can answer questions 2 and 3 for each bridge.

Group 1's bridge

Weight of bridge:

Weight the bridge held:

Bridge efficiency ratio:

Where did the bridge break? Did it fail at a toothpick or marshmallow?

Continue for each group.....

Note to teachers:

The bridges tend to shatter, breaking quickly, so it can be hard to answer the above questions. Depending on the design, the marshmallows tend to break before the toothpicks. Also, depending on the design, the weakest point of the bridge (typically) is where the towers meet the roadway. The downward force on the bridge tends to make the towers want to push outwards. This causes a twisting force on the bridge, concentrated where the span just starts to leave the bridge tower.

Also, it will be surprising how strong the bridges will be. I had bridge ratios ranging from 19.6 to 135.4. The strongest bridge held almost 19 pounds. The bridges weighed from 1.6 ounces to 2.7 ounces.